Maharaja College Ara B.Sc Part-I

Group-B Mathematics

Def: Symmetric Matrices - A matrix 'A' is Symmetric if $A^T = A$ i.e if for all the values i and j $a_{ij} = a_{ji}$

E.g. Let $A = \begin{bmatrix} 2 & -3 & 5 \\ -3 & 6 & 7 \\ 5 & 7 & -8 \end{bmatrix}$

$$a_{12} = 3 = a_{21}, \quad a_{13} = 5 = a_{31}, \quad a_{23} = 7 = a_{32}$$

So, here $A^T = \begin{bmatrix} 2 & -3 & 5 \\ -3 & 6 & 7 \\ 5 & 7 & -8 \end{bmatrix} = A$

$\Rightarrow$ So, A is a Symmetric matrix

Skew Symmetric Matrix - A Square matrix A is said to be skew Symmetric i.e $A^T = -A$ i.e. if for all the value of i and j, $a_{ij} = -a_{ji}$

Clearly. The diagonal elements of such a matrix must be zero because

$$a_{ii} = -a_{ii} \Rightarrow a_{ii} = 0$$

Eg. $\begin{bmatrix} 0 & -h & g \\ h & 0 & -f \\ g & f & 0 \end{bmatrix}$ is a skew Symmetric matrix.

$$B = \begin{bmatrix} 0 & -3 & 4 \\ 3 & 0 & 5 \\ 4 & -5 & 0 \end{bmatrix}$$

The diagonal elements of B are zero and $a_{ij} = -a_{ij}$. So it is skew symmetric matrix.

Orthogonal Matrix A real matrix A is Orthogonal if $A^T = A^{-1}$ that is if $AA^T = A^TA = I$. Thus A must necessarily be square and invertible where I is unit Matrix.

Eg. Let $A = \begin{bmatrix} 1/9 & 8/9 & -4/9 \\ \frac{4}{9} & -\frac{4}{9} & -7/9 \\ 8/9 & 1/9 & 4/9 \end{bmatrix}$. Here Multiplying A by A^T yields I that is $AA^T = I$. This means $A^TA = I$, as well.

Thus $A^T = \cancel{A^T}\ A^{-1}$, that is A is Orthogonal

Note: Suppose A is a real orthogonal 3×3 matrix with rows $u_1 = (a_1, a_2, a_3)$ $u_2 = (b_1, b_2, b_3)$ & $u_3 = (c_1, c_2, c_3)$

Because A is Orthogonal, we must have $AA^T = I$.

$\Rightarrow a_1^2 + a_2^2 + a_3^2 = 1$

$\Rightarrow b_1^2 + b_2^2 + b_3^2 = 1$

$\Rightarrow c_1^2 + c_2^2 + c_3^2 = 1$

And

$a_1b_1 + a_2b_2 + a_3b_3 = 0$

$b_1a_1 + b_2a_2 + b_3a_3 = 0$

$b_1c_1 + b_2c_2 + b_3c_3 = 0$

$c_1b_1 + c_2b_2 + c_3b_3 = 0$

$a_1c_1 + a_2c_2 + a_3c_3 = 0$

$c_1a_1 + c_2a_2 + c_3a_3 = 0$